**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:** **B.**

We have a normal distribution with mean= 45 and Sd= 8

Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find P(X > 50).

P(X > 50) = 1 - P(X ≤ 50).

Z = (X - μ)/σ

= (X - 45)/8

Thus the question can be answered by using the normal table to find

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0)

= P(Z ≤ 0.625)

=73.4%

Probability that the service manager will not meet his demand will be =

100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans:** We have normal distribution with mean = 38 and Standard Deviation = 6.

Let as assume X be the number of employees.

Then Probability of employees greater than age of 44 = P(X>44)

P(X>44) = 1 – P(X<= 44)

To find Z,

Z = (X - μ)/σ

= (X – 38)/6

P(X<=44) = P(Z<= (44 – 38)/6)

= P(Z<= 1) = 0.8413 = 84.13%

Probability that the employee will be greater than age of 44 = 100 – 84.13

=15.87 %

Probability of employees between 38 &44 = P(X<=44) – P(X> = 38)

P(X>=38) = P(Z>=(38-38)/6)

= P(Z>=0) = 0.5

Therefore,

P(X<=44) – P(X> = 38) = 0.8413 – 0.5 = 0.3413 = 34.13 %

So the statement ‘More employees at the processing center are older than 44 than between 38 and 44’ is **TRUE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:** Probability of employees less than age of 30 = P(X<30)

Z = (X -μ)/σ

=(30-38)/6 = -1.333

P(X<30) =P(Z<=-1.333) = 9.18

So, the number of employees with probability 0.918 of them being under age 30 =

0.0918\*400=36.72(or 36 employees).

Hence the statement B is **TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:** As we know that if , X ∼ N(μ1, σ1^2 ), and Y ∼ N(μ2, σ2^2 ) are two independent random

variables then,

X + Y ∼ N(μ1 + μ2, σ1^2 + σ2^2 ) , and

X − Y ∼ N(μ1 − μ2, σ1^2 + σ2^2 ) .

Similarly, if

Z = aX + bY, where X and Y are as defined above, i.e. Z is linear combination an Y,

Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question

2X1~ N(2 u,4 σ^2) and

X1+X2 ~ N(μ + μ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )

2X1-(X1+X2) = N( 4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans:** Given that the Probability of getting value between a and b should be 0.99.

Mean = 100 ,Standard Deviation =20

So the Probability of going wrong is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

Z = (X- μ) / σ

Using Python,

Z value is given as stats.norm.ppf(pvalue)

Z value at 0.5th percentile is given as

                          Z(0.5) = stats.norm.ppf(0.005)= -2.576

Z value at 99.5 percentile is given as

                         Z(99.5) = stats.norm.ppf(0.995) = 2.576

Z \* σ + μ = X

Z(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

Please check Ans\_5.